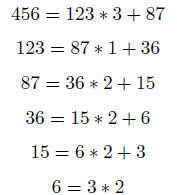
**Homework 3 Solution**

1. Thinking like an instructor: Make up a solvable Diophantine equation. Explain what process you used and which results you relied on during the process.

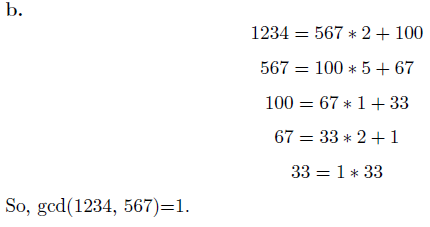
3x + 369633636969691y = 400 and 5x + 1432175741y = 36

Basically I chose my coefficients to have a GCD of 1 and set it equal to literally any random number I wanted and it's possible.

I created two numbers with a common, slightly large gcd by using the prime numbers less than 100 list. I chose *p*=89, and let I also chose one of the multipliers to be a bit more than twice the other multiplier so that in the Euclidean algorithm, the remainder will be small. If I chose 83 instead of 97, the remainder will not be much smaller than *a* in the first step, which helps keep the number of steps low.Then if I choose the right hand side a multiple of the gcd, I know there’s a solution, so say

1. Use Euclidean algorithm to find the gcd of the following pairs:   
   a.





1. What are the possible values of? If you guess a possible value, make sure that it occurs.

For any *n,* , so anything which divides both numbers divides 3. This means gcd can be 1 or 3. For *n*=1, gcd(28,5)=1, and for *n*=3, gcd(120,9)=3. So both values occur.

* so, by calculating remainders,
* .
* if and only if is not a multiple of 3.
  + This is because if is not a multiple of 3 then there exists such that which using the as a linear combination means that .
  + Conversely, if is a multiple of 3 then and will share a factor of at least 3 and their will be at least 3, and thus not 1.

1. a. Find an integer solution to the equation.

Through the Division/Euclidean Algorithm, we can get

561 ( -177 ) + 379 ( 262 ) = 1

b. Find a second solution with *x>*0.

To do this we will multiply our whole last equation by -378. This will give us

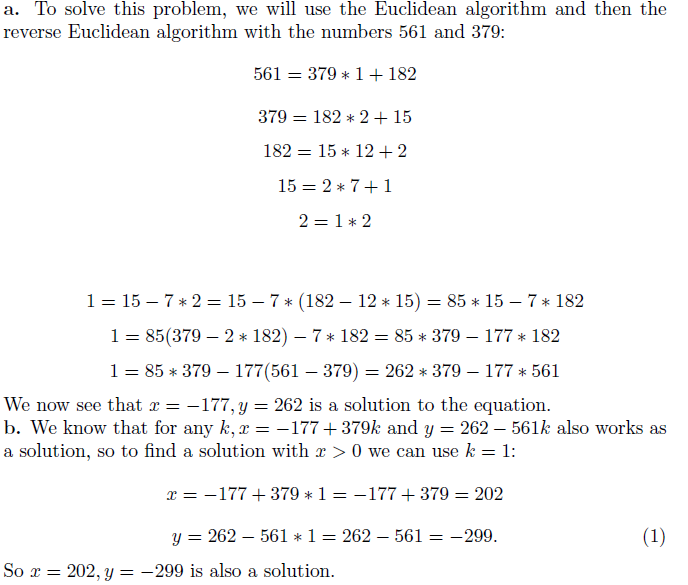
561 \* (66906) + 379 (-99036) = -378

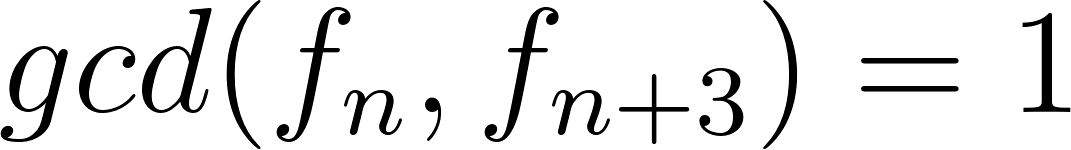
Next we add 379 to both sides to get

561 \* (66906) + 379 (-99036+1) = -378 +379

So

561 \* (66906) + 379 (-99035) = 1



1. Show that [](https://www.codecogs.com/eqnedit.php?latex=gcd(f_n%2C%20f_%7Bn%2B3%7D)%3D1#0) or 2.

Since f\_n+3 = 4 f\_n + f\_n-3 for n>6 Notice this is essentially the Euclidean algorithm. This means that GCD ( f\_n+3 , f\_n ) = GCD ( f\_n , f\_n-3 ). Therefore, continuing in this way, we can reduce each GCD ( f\_n+3 , f\_n ) to one of the base cases of GCD (f\_3, f\_0), GCD (f\_4, f\_1) or GCD (f\_5, f\_2). Note that

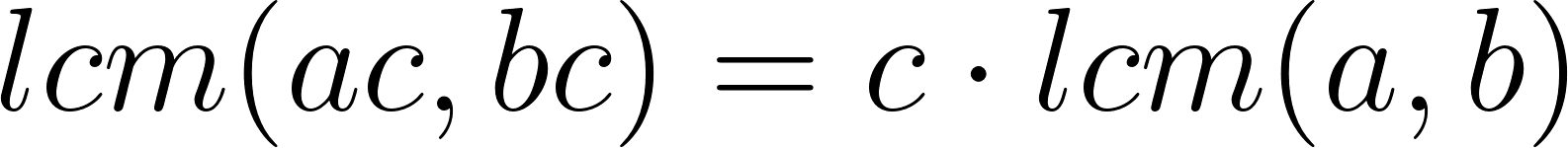
GCD (f\_3, f\_0) = GCD (2, 0) = 2

GCD (f\_4, f\_1) = GCD (3, 1) = 1

GCD (f\_5, f\_2) = GCD (5, 1) = 1

So GCD ( f\_n+3 , f\_n ) is 1 or 2 depending on if n is divisible by 3.

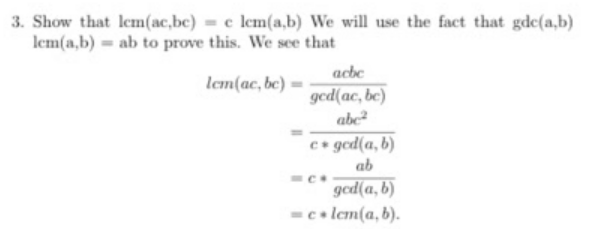
Suppose and Then So and If is even, consider Then and If is not even, then In either case, we have a factor that divides both and This means that this factor also divides Continuing in this way, we see that this factor divides So either or Note also that and So both values occur.

1. Prove that if *c>*0, then [](https://www.codecogs.com/eqnedit.php?latex=lcm(ac%2Cbc)%3Dc%5Ccdot%20lcm(a%2Cb)#0).

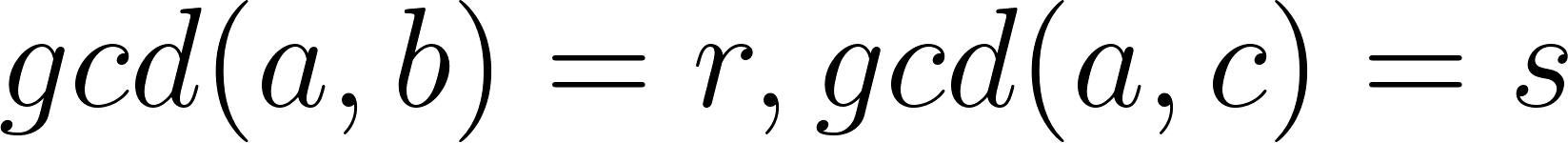
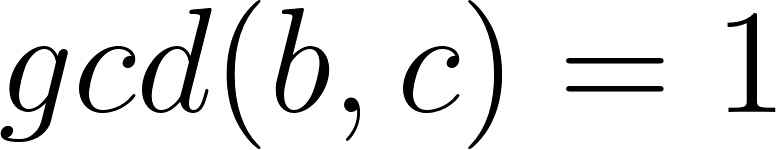
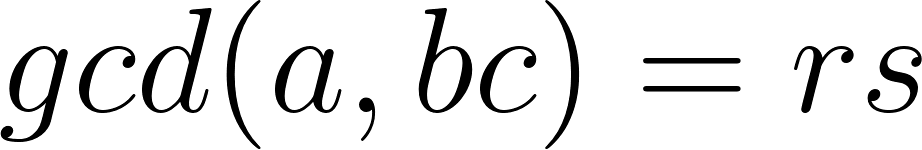
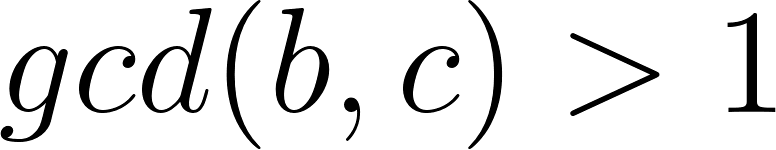
By definition of lcm, and so both and divide and hence is a common multiple of and . Therefore (1)

By definition of lcm, and So and This means that is a common multiple of and . Therefore . (2)

Combining (1) and (2) gives us that .



Note: For this method, we also have to separate the case for when *a* or *b*=0 and also “without loss of generality” assume a, b>0 because lcm(a,b)=lcm(|a|, |b|), but these are pretty straightforward and I don’t have the source for this solution :)

1. Show that if [](https://www.codecogs.com/eqnedit.php?latex=gcd(a%2Cb)%3Dr%20%2C%20gcd(a%2Cc)%3Ds#0) and [](https://www.codecogs.com/eqnedit.php?latex=gcd(b%2Cc)%3D1#0), then [](https://www.codecogs.com/eqnedit.php?latex=gcd(a%2Cbc)%3Drs#0). Give an example to show that this need not be true if [](https://www.codecogs.com/eqnedit.php?latex=gcd(b%2Cc)%3E1#0).

We will use Theorem 4 (from Week 2 activity) characterization of the gcd to prove So we need to show:   
 (which is obvious)

and :

The latter is obvious, and the first follows from Theorem 5 of Week 2 activity. What we want to show is equivalent to because we can multiply both sides by To show , note that and because Therefore, by Theorem 5, .

3) For every common divisor *d* of *a* and *bc,* :

Suppose and Let Then and so Now consider If we can show , we are done. Note that so Also, and so by Theorem 5. This means Hence, divides

Therefore, is equal to the gcd of *a* and *bc.*

The gcd equality is not true if we consider the following example:   
, and

We will show that by showing and First note that because So since each divides *a* and they are relatively prime (second corollary to Theorem 5 in Week 2 class activity). Also because each gcd divides *b* and *c,* respectively. So divides both *a* and *bc*, which means . To show the other inequality, consider Let Then Note that the remaining part of *d,* i.e. and But so by Theorem 5 (Ibid.) Therefore, Then This proves the other inequality, and finishes the proof.

Hmm.. why reinvent the wheel? I decided not to do a third solution and searched for it online instead. Here are two possible solutions (the first one uses half of my first solution above):  
<https://math.stackexchange.com/questions/536179/prove-that-if-gcda-b-1-then-gcdab-c-gcda-c-gcdb-c?rq=1>

<https://math.stackexchange.com/questions/1492022/does-gcda-bc-divides-gcda-b-gcda-c/1492083>

(The second answer in this second link is another solution, but it seems to have a few extra steps.)

1. Write a code to implement the Euclidean algorithm to find gcd.

